7.3 The analysis of a tidal power facility is similar to that for a normal wind turbine. That is, we can still write $P = 1/2\,\rho A v^3$ but now $\rho = 1000$ kg/m^3 and v is the speed of water rushing toward the turbine. The following graphs assume sinusoidally varying water speed, with amplitude V_{max}. We assume the turbine can accept flows in either direction (as the tide ebbs and floods) so it is only the magnitude of the tidal current that matters.

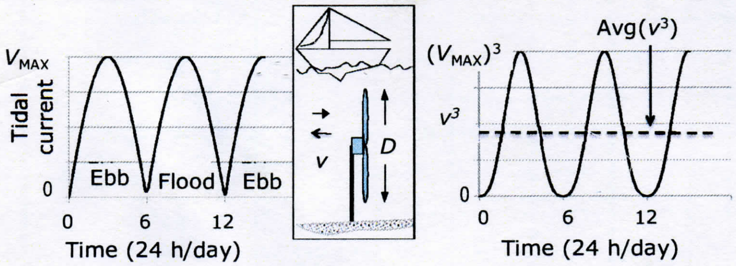


Figure P 7.3

For a sinusoidal tidal flow with $V_{max} = 2$ m/s,

a. What is the average power density (W/m^2) in the tidal current? A bit of calculus gives us the following helpful start:

$$\left(v^3\right)_{avg} = avg\left(V_{max}\sin v\right)^3 = V_{max}^3\frac{\int_0^{\pi/2}\sin^3 v\,dv}{\pi/2} = \frac{4}{3\pi}V_{max}^3$$

b. If a 600-kW turbine with 20-m diameter blades has a system efficiency of 30%, how many kWh would it deliver per year in these tides?

a) $\frac{P}{A} = \frac{1}{2}\rho\, V_{AVG}^3 = \rho\, \frac{4}{3\pi} V_{MAX}^3$

$= \frac{1000}{2}\ \frac{4}{3\pi}\ (2)^3$

$= 1698\ \frac{W}{m^2}$

b) $A = \pi r^2 = \pi\left(\frac{D}{2}\right)^2 = \pi(10)^2 = \underline{\underline{314.1}}\ m^2$ ANS

$P_{WIND} = \frac{\rho}{2} A\, V_{AVG}^3 = (314.1)(1.698)\ \frac{kW}{m^2}$

$= 553.3\ kW$

$=$

$P_{TURB} = \eta\, P_{WIND} = .3(553.3 kW) = 165.99$

$E = P_{TURB} \cdot 365\ DAYS \cdot \frac{24\ HR}{DAY}$

$= \underline{\underline{1,454\ \frac{MWh}{YR}}}$ ANS.

7.4 Consider an 82-m (diameter), 1.65-MW wind turbine with a rated wind speed of 13 m/s.

a. At what rpm does the rotor turn when it operates with a TSR of 4.8 in 13 m/s winds? How many seconds per rotation is that?

b. What is the tip speed of the rotor in those winds (m/s and mph)?

c. What gear ratio is needed to match the rotor speed to an 1800 rpm generator when the wind is blowing at the rated wind speed?

d. What is the efficiency of the complete wind turbine (blades, gearbox, generator) in 13 m/s winds? (Masters 493-494)

a) $TSR = \frac{\text{BLADE TIP SPEED}}{\text{WIND SPEED}} = \frac{BTS}{WS}$

$BTS = TSR \cdot WS = 4.8 \cdot 13\ m/s = 62.4\ \frac{m}{s}$

BLADE SWEEP CIRCUMFERECE = $\pi D = 3.14 \cdot 82 = 257.6\ M$

$t_{1\,REV} = \frac{257.6\ m}{62.4\ \frac{m}{s}} = 4.12\ \frac{s}{REV}$

$\Rightarrow \underline{\underline{.24}}\ \frac{ROTATIONS}{S}$ ANS

$RPM = .24\ \frac{ROTATIONS}{S} \cdot \frac{60\,S}{MIN} = \underline{\underline{14.56\ RPM}}$ ANS.

b) BLADE TIPSPEED(BTS) = $\underline{\underline{62.4\ m/s}}$ ANS.

$BTS = 62.4\ \frac{m}{s} \left| \frac{KM}{1000\,M} \right| \frac{.623\ mi}{1\ KM} \left| \frac{3600\ S}{HR} \right.$

$= \underline{\underline{139.9\ MPH}}$ ANS.

c) GEAR RATIO $\frac{MOTOR}{TURBINE\ SHAFT} = \frac{1800}{14.56} \Rightarrow \underline{\underline{123.63 : 1}}$ ANS.

d) RATED PWR @ 13 m/s = 1.65 MW

$P_{WIND} = \frac{\rho}{2} A v^3 = 1.225 \left(\underbrace{\pi \left(\frac{82}{2}\right)^2}_{\text{BLADE SWEEP AREA}} \right) \cdot (13)^3$

$= 7,106\ MW$

$\eta = \frac{P_{TURBINE}}{P_{WIND}} = \frac{1.65\ MW}{7,106\ MW} \times 100\% = \underline{\underline{23.2\%}}$ ANS EFFICIENT.

7.5 An early prototype 10-kW Makani Windpower system consisted of two 5-kW wind turbines mounted on a wing that flies in somewhat vertical circles (like a kite) several hundred meters above ground. A tether attached to the "kite" carries power from the turbines down to the ground. Since the speed of the kite-turbines moving through the air is much faster than the wind speed, much smaller turbine blades can be used than those on conventional ground-mounted wind turbines. Also with no need for a tower, the cost of materials is far lower than for a conventional system.

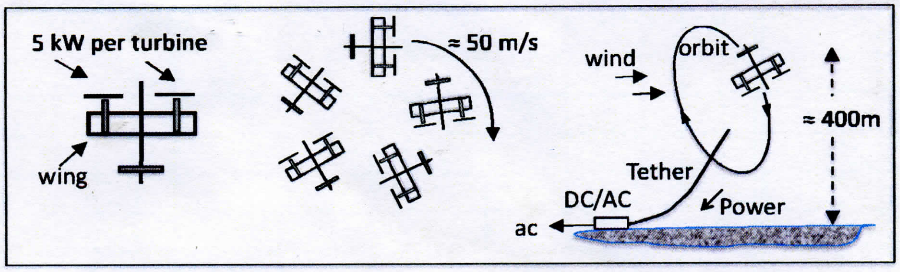


Figure P 7.5

Suppose each wing/turbine is moving through the air at 50 m/s and suppose the overall efficiency is half that of the Betz limit, what blade diameter would be required to deliver 5 kW of power per turbine. Don't bother to correct air density for this altitude.

$$P = \frac{1}{2}\rho A v^3 \cdot C_p$$

; v GIVEN AS 50 M/S

$P = 5\text{ kW}$

$A = \pi r^2 \Rightarrow r^2 = \frac{A}{\pi}$

$\rho = 1.225 \frac{kg}{m^3}$

$C_p \approx .5\underbrace{(.59)}_{\text{BETZ LIMIT}} \text{ (FIG 7.18)}$

$= .295$

$$r^2 = \frac{2P}{\rho \pi v^3 \cdot .295}$$

$$= \frac{2(5000)}{1.225\,\pi\,(50)^3 .295}$$

$$r^2 = .0705$$

$$r = .265 \text{ M}$$

ANS.

$\Rightarrow$ BLADE DIAMETER = .53 M ANS.

$$\frac{53\text{ cm}}{} \Big| \frac{1\text{ in}}{2.54\text{ cm}} \Big| \frac{\text{ft}}{12\text{ in}}$$

$= 1.74$ ft

7.6 Consider the following probability density function for wind speed:

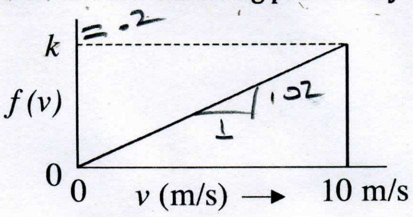


Figure P 7.6

a. What is an appropriate value of k for this to be a legitimate pdf?

b. What is the average power in these winds (W/m²) under standard temperature and pressure conditions (1 atm, 15°C)?

a) FOR A PROB. DENSITY FUNC, THE AREA UNDER THE CURVE MUST BE 1 ⇒

$$CDF = \int_0^x pdf\, dt = 1 \quad x \to \infty$$

$$\therefore \quad 1 = \frac{1}{2}(10)k$$

$$k = \frac{1}{5} = \underline{\underline{.2}} \text{ ANS.}$$

b) $$(v^3)_{AVG} = \int_0^\infty v^3 f(v)\, dv \leftarrow \text{PROBABILITY DENSITY FUNC.}$$

$$= \int_0^{10} v^3(.02v)\, dv$$

$$= .02 \left.\frac{v^5}{5}\right|_0^{10} = \frac{.2}{5} 10^5 = 400$$

$$\frac{P_{AVG}}{A} = \frac{1}{2}\rho\, (v^3)_{AVG} = \frac{1}{2}(1.225)(400)$$

$$= \underline{\underline{245 \frac{W}{m^2}}} \Big\} \text{ SPECIFIC AVERAGE POWER @ 1 ATM, 15°C}$$

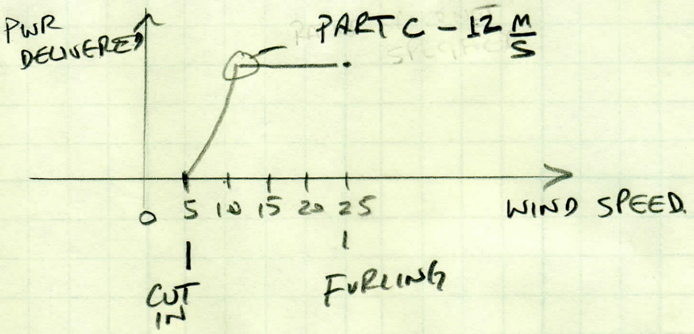
ANS.

7.7 Suppose a wind turbine has a cut-in wind speed of 5 m/s and a furling wind speed of 25 m/s. If the winds the turbine sees have Rayleigh statistics with an average wind speed of 9 m/s,

a. For how many hours per year will the turbine be shut down because of excessively high-speed winds?

b. For how many hours per year will the turbine be shut down because winds are too low?

c. If this is a 1-MW turbine, how much energy (kWh/yr) would be produced for winds blowing at or above the rated wind speed of 12 m/s?



a) EXCESSIVELY HIGH WINDS

$\Rightarrow v > 25 \frac{m}{s}$

$$P(v > 25) = 1 - P(v \le 25)$$
$$= 1 - \left[1 - e^{-\frac{\pi}{4}\left(\frac{v}{\bar{v}}\right)^2}\right]$$
$$= e^{-\pi/4\left(\frac{25}{9}\right)^2}$$
$$= .0023$$

EXPECTED HOURS SHUTDOWN/YR

$$= .0023(8760) = \underline{\underline{20.15 \text{ HOURS}}}$$ ANS.

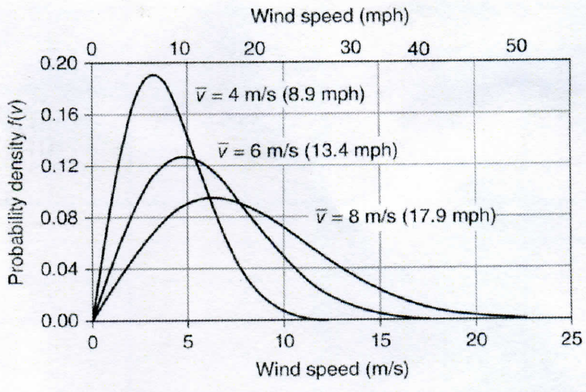


Fig 7.27 Rayleigh pdf w/ varying wind speeds

b) WINDS BELOW 5 m/s CUT-IN

$$P(v < 5 \text{ m/s}) = 1 - e^{-\frac{\pi}{4}\left(\frac{5}{9}\right)^2} = 1 - .7847 = .2152$$

EXPECTED HRS $(v < 5/\text{ms}) = .2152(8760) = \underline{\underline{1885.1}}$ HOURS ANS

c) $P(12 \le v \le 25)$

$$P(v \ge 12) = 1 - P(v \le 12) = 1 - \left(1 - e^{-\frac{\pi}{4}\left(\frac{12}{9}\right)^2}\right)$$
$$= .2475$$

$$P(12 \le v \le 25) = .2475 - .0023 = .24501$$

EXPECTED ENERGY

$\Rightarrow 1\text{MW} \cdot .2475 \cdot 8760$

$= \underline{\underline{2148 \text{ MWh}}}$ ANS.

7.8 The table below shows a portion of a discretized estimate of the energy delivered by a Siemens 2300-kW, 101-m diameter wind turbine (Table 7.5) exposed to Rayleigh winds with average speed 6 m/s. For example, for winds blowing around 4 m/s ($3.5 \le v \le 4.5$), the turbine produces 100 kW of power and in a year's time, a Rayleigh pdf predicts it delivers 107,841 kWh.

a. How many kWh/yr would be generated for winds blowing at 5 m/s winds? Do this by hand.

b. Create your own spreadsheet (similar to Example 7.9) and find the annual energy delivered by this turbine in these winds.

c. Compare your answer in (b) to the energy predicted by using the simplified capacity factor Equation 7.63.

P (MW)	2300	
v avg	6	m/s
v (m/s)	kW	kWh/y
0	0	–
1	0	–
2	0	–
3	0	–
4	100	107,84
5	?	?
etc		

FIGURE P7.8

Masters Problem 7.8 (c) Solution

Turbine Sweep Area	8011.8467	m^2
Prated	2300	MW
v_avg	6	m/s
cut-in speed	4	m/s

		prob density function range					
	v (m/s)	low V	hi V	p(low V <= v <= hi V)	h/yr	kW	KWh/yr
Below Cut-in speed	0	0	0.5	5.4E-03	47.6	0	0
	1	0.5	1.5	4.2E-02	372.0	0	0
	2	1.5	2.5	8.0E-02	697.0	0	0
	3	2.5	3.5	1.1E-01	937.8	0	0
Transition Cut-in to rated speed	4	3.5	4.5	1.2E-01	1073.9	100	107391
	5	4.5	5.5	1.3E-01	1103.9	230	253887
	6	5.5	6.5	1.2E-01	1042.9	420	438025
	7	6.5	7.5	1.0E-01	917.2	720	660394
	8	7.5	8.5	8.6E-02	756.6	1100	832240
	9	8.5	9.5	6.7E-02	588.2	1530	899937
	10	9.5	10.5	4.9E-02	432.4	2000	864846
	11	10.5	11.5	3.4E-02	301.3	2240	674993
P_{rated}	12	11.5	12.5	2.3E-02	199.4	2300	458601
	13	12.5	13.5	1.4E-02	125.4	2300	288525
	14	13.5	14.5	8.6E-03	75.1	2300	172772
	15	14.5	15.5	4.9E-03	42.8	2300	98550
	16	15.5	16.5	2.7E-03	23.3	2300	53583
	17	16.5	17.5	1.4E-03	12.1	2300	27785
	18	17.5	18.5	6.8E-04	6.0	2300	13747
	19	18.5	19.5	3.2E-04	2.8	2300	6492
	20	19.5	20.5	1.5E-04	1.3	2300	2927
	21	20.5	21.5	6.3E-05	0.5	2300	1261
	22	21.5	22.5	2.6E-05	0.2	2300	519
	23	22.5	23.5	1.0E-05	0.1	2300	204
	24	23.5	24.5	3.8E-06	0.0	2300	77
	25	24.5	25.5	1.4E-06	0.0	2300	27
	26	25.5	26.5	4.7E-07	0.0	0	0
			Cum Dist =	1.000	Cum Hours / year = 8759.998		

Masters Problem 7.8 (c) Solution